SUATU KAJIAN PADA BCI-ALJABAR YANG TERKAIT DENGAN SIFAT DERIVASI KIRI

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ABSTRAK. Setiap (α,β)-Derivasi kiri pada *BCI*- Aljabar X dapat dikatakan regular jika hasil derivasinya sama dengan nol atau setiap X idealnya merupakan *D*-invarian. Suatu himpunan A dikatakan ideal pada X dengan α-ideal jika $\alpha(A) \subseteq A$, hal serupa dengan (β-ideal jika $\beta(A) \subseteq A$). Selain itu, himpunan A dikatakan ideal pada X dengan *D*-invarian jika$D(A) \subseteq A$. (α,β)- Derivasi kiri *BCI*- Aljabar X positif semisederhana merupakan hasil operasi biner antara endomorfisma dengan derivasinya. Himpunan X dikatakan torsi bebas *BCI*-aljabar jika kuadrat derivasinya atau komposisi derivasi pertama dan derivasi kedua sama dengan nol.
Kata kunci :BCI-aljabar, BCI-aljabar p-semi sederhana, (α,β)-Derivasi kiri,
D-invarian

I. PENDAHULUAN

Suatu struktur aljabar atau sistem matematika merupakan himpunan yang tidak kosong dengan paling sedikit sebuah relasi ekuivalensi, satu atau lebih operasi biner dengan aksioma-aksioma tertentu.

Misalkan $(X,\bullet)$ suatu grup dengan operasi biner "$\bullet$" dan e unsur identitas dari X. Jika pada X dilengkapi operasi biner "$\odot$" serta memenuhi aksioma-aksioma tertentu maka akan membentuk struktur aljabar baru yang dinamakan *K*-aljabar. Jika operasi "$\odot$" didefinisikan oleh $x \odot y = x \bullet y^{-1}$, operasi "$\odot$" bersifat tertutup pada X atau operasi "$\odot$" merupakan operasi biner. $\forall x, y \in X$ maka operasi "$\odot$" bersifat tertutup pada X atau operasi "$\odot$" merupakan operasi biner.

II. HASIL DAN PEMBAHASAN

Definisi 2.1 [12]

Suatu himpunan tak kosong X dengan operasi biner $*$ dan elemen khusus 0 disebut *BCI*-Aljabar jika untuk setiap $x, y, z \in X$ memenuhi aksioma-aksioma berikut.

I. $\big((x*y)*(x*z)\big)*(z*y)=0$

II. $\big(x*(x*y)\big)*y=0$

III. $x*x=0$

IV. $x*y=0$ dan $y*x=0$ sehingga $x=y$

Definisi 2.2 [10]

Misal X adalah *BCI*-Aljabar . *Self map* $d_{(\alpha,\beta)}: X \to X$ disebut $(\boldsymbol{\alpha},\boldsymbol{\beta})$**-Derivasi** pada X jika memenuhi:

$$(\forall x, y \epsilon X)\left(d_{(\alpha,\beta)}(x * y)\right) = \left(d_{(\alpha,\beta)}(x) * \alpha(y)\right) \wedge \left(d_{(\alpha,\beta)}(y) * \beta(x)\right)$$

Definisi 2.3 [1]

Misal X adalah *BCI*-Aljabar. *Self map* $D: X \to X$ disebut Derivasi kiri pada X jika memenuhi:

$$\left(D(x * y)\right) = (x * D(y)) \wedge (y * D(x))\ ;\ (\forall x, y \epsilon X)$$

Proposisi 2.4 [1]

Misal D adalah derivasi kiri pada *BCI*-Aljabar X, dan $\forall\ x, y \in X$, sehingga didapat

(1) $x * D(x) = y * D(y)$

(2) $D(x) = a_{D(x) \wedge x}$

(3) $D(x) = D(x) \wedge x$

(4) $D(x) \in L_P(X)$

Bukti:

(1) Misal $x, y \in X$

$D(0) = D(x * x)$

$= \left(x * D(x)\right) \wedge \left(x * D(x)\right)$

$= (x * D(x))$

dan

$D(0) = D(y * y)$

$= \left(y * D(y)\right) \wedge \left(x * D(y)\right)$

$= (y * D(y))$

Sehingga dapat disimpulkan $x * D(x) = y * D(y)$

(2),(3),(4) dapat dibuktikan sendiri

Definisi 2.5 [1]

Diberikan X suatu *BCI*-Aljabar.Dengan Derivasi kiri pada X, mengartikan suatu *self-map D* terhadap X yang memenuhi:

$$(\forall x, y \epsilon X)\left(D(x * y)\right) = (x * D(y)) \wedge (y * D(x))$$

Untuk semua $x, y \in X$

Proposisi 2.6 [11]

Misal $d_{(\alpha,\beta)}$adalah (α,β)-Derivasi kiri pada *BCI*-Aljabar X, maka

(1) $(\forall x \in X)\left(x \in L_P(X) \to d_{(\alpha,\beta)}(x) \in L_P(X)\right)$

(2) $\left(\forall x \in L_P(X)\right)\left(d_{(\alpha,\beta)}(x) = 0 + d_{(\alpha,\beta)}(x)\right)$

(3) $\left(\forall x \in L_P(X)\right)\left(d_{(\alpha,\beta)}(x + y) = \alpha(x) + d_{(\alpha,\beta)}(y)\right)$

(4) $(\forall x \in X)\left(x \in G(X) \to d_{(\alpha,\beta)}(x) \in G(X)\right)$

Bukti:

(1) $\forall x \in L_P(X)$

$$
\begin{aligned}
d_{(\alpha,\beta)}(x) &= d_{(\alpha,\beta)}\left(0 * (0 * x)\right) \\
&= \left(\alpha(0) * d_{(\alpha,\beta)}(0 * x)\right) \wedge \left(\beta(0 * x) * d_{(\alpha,\beta)}(0)\right) \\
&= \left(0 * d_{(\alpha,\beta)}(0 * x)\right) \wedge \left(\beta(0 * x) * d_{(\alpha,\beta)}(0)\right) \\
&= \left(\beta(0 * x) * d_{(\alpha,\beta)}(0)\right) * \left(\left(\beta(0 * x) * d_{(\alpha,\beta)}(0)\right) * \left(0 * d_{(\alpha,\beta)}(0 * x)\right)\right) \\
&= 0 * d_{(\alpha,\beta)}(0 * x) \in L_P(X)
\end{aligned}
$$

(2),(3),(4) dapat dibuktikan sendiri

Proposisi 2.7 [11]

Misal $d_{(\alpha,\beta)}$ adalah (α,β)-Derivasi kiri pada *BCI*-Aljabar X, maka

(1) $(\forall x \in X)\left(x \in L_P(X) \to d_{(\alpha,\beta)}(x) = \alpha(x) + d_{(\alpha,\beta)}(0)\right)$

(2) $\left(\forall x \in L_P(X)\right)\left(d_{(\alpha,\beta)}(x + y) = d_{(\alpha,\beta)}(x) + d_{(\alpha,\beta)}(y) - d_{(\alpha,\beta)}(0)\right)$

(3) $(\forall x, y \in X)\left(d_{(\alpha,\beta)}(x + y) \leq \alpha(x) * d_{(\alpha,\beta)}(y)\right)$

(4) Jika α adalah pemetaan identitas pada X, maka $d_{(\alpha,\beta)}$identitas pada $L_P(X)$ jika dan hanya jika $d_{(\alpha,\beta)}(0) = 0$

Bukti:

(1) $\forall x \in L_P(X)$, maka kita mempunyai

$$
\begin{aligned}
d_{(\alpha,\beta)}(x) &= d_{(\alpha,\beta)}(x * 0) \\
&= \left(\alpha(x) * d_{(\alpha,\beta)}(0)\right) \wedge \left(\beta(0) * d_{(\alpha,\beta)}(x)\right) \\
&= \left(\alpha(x) * d_{(\alpha,\beta)}(0)\right) \wedge \left(0 * d_{(\alpha,\beta)}(x)\right) \\
&= \left(0 * d_{(\alpha,\beta)}(x)\right) * \left(\left(0 * d_{(\alpha,\beta)}(x)\right) * \left(\alpha(x) * d_{(\alpha,\beta)}(0)\right)\right) \\
&= \left(0 * d_{(\alpha,\beta)}(x)\right) * \left(\left(0 * \left(\alpha(x) * d_{(\alpha,\beta)}(0)\right)\right) * d_{(\alpha,\beta)}(x)\right) \\
&= 0 * \left(0 * \left(\alpha(x) * d_{(\alpha,\beta)}(0)\right)\right) \\
&= \alpha(x) * d_{(\alpha,\beta)}(0) \\
&= \alpha(x) * \left(0 * d_{(\alpha,\beta)}(0)\right) \\
&= \alpha(x) + d_{(\alpha,\beta)}(0)
\end{aligned}
$$

Sejak $\alpha(x) * d_{(\alpha,\beta)}(0) \in L_P(X)$ dan $d_{(\alpha,\beta)}(0) \in G(X)$

(2) Jika $x, y \in L_P(X)$, maka $x + y \in L_P(X)$. Dengan menggunakan (1) diperoleh

$$\begin{aligned} d_{(\alpha,\beta)}(x+y) &= \alpha(x+y) + d_{(\alpha,\beta)}(0) \\ &= \alpha(x) + \alpha(y) + d_{(\alpha,\beta)}(0) \\ &= \alpha(x) + d_{(\alpha,\beta)}(0) + \alpha(y) + d_{(\alpha,\beta)}(0) - d_{(\alpha,\beta)}(0) \\ &= d_{(\alpha,\beta)}(x) + d_{(\alpha,\beta)}(y) - d_{(\alpha,\beta)}(0) \end{aligned}$$

(3),(4) dapat dibuktikan sendiri

Proposisi 2.8 [11]

Misal $d_{(\alpha,\beta)}$adalah (α,β)-Derivasi kiri pada *BCI*-Aljabar X p-semi sederhana, maka $(\forall x, y \in X)\left(d_{(\alpha,\beta)}(x * y) = \alpha(x) * d_{(\alpha,\beta)}(y)\right)$.

Bukti:

Misal X p-semi sederhana *BCI*-Aljabar, dan $\forall x, y \in X$, diperoleh

$$\begin{aligned} d_{(\alpha,\beta)}(x * y) &= \left(\alpha(x) * d_{(\alpha,\beta)}(y)\right) \wedge \left(\beta(y) * d_{(\alpha,\beta)}(x)\right) \\ &= \left((\beta(y) * d_{(\alpha,\beta)}(x)\right) * \left(\left(\beta(y) * d_{(\alpha,\beta)}(x)\right) * \left(\alpha(x) * d_{(\alpha,\beta)}(y)\right)\right) \\ &= \alpha(x) * d_{(\alpha,\beta)}(y) \end{aligned}$$

Proposisi 2.9 [11]

Misal $d_{(\alpha,\beta)}$adalah (α,β)-Derivasi kiri pada *BCI*-Aljabar X. Jika ${d^2}_{(\alpha,\beta)} = 0$ pada $L_P(X)$, maka berlaku

$$\left(\alpha \circ d_{(\alpha,\beta)}\right)(x) = \frac{1}{2}\left(\left(\alpha \circ d_{(\alpha,\beta)}\right)(0) - d_{(\alpha,\beta)}(0)\right);\ \forall x \in L_P(X)$$

Bukti :

Asumsikan ${d^2}_{(\alpha,\beta)} = 0$ pada $L_P(X)$. Jika $x \in L_P(X)$ maka $x + x \in L_P(X)$, sehingga dengan menggunakan proposisi 3.3.1 (1), proposisi 3.3.2 (1) dan (2) diperoleh

$$\begin{aligned} 0 &= {d^2}_{(\alpha,\beta)}(x+x) \\ &= d_{(\alpha,\beta)}\left(d_{(\alpha,\beta)}(x+x)\right) \\ &= d_{(\alpha,\beta)}(0) + \alpha\left(d_{(\alpha,\beta)}(x+x)\right) \\ &= d_{(\alpha,\beta)}(0) + \alpha\left(d_{(\alpha,\beta)}(x) + d_{(\alpha,\beta)}(x) - d_{(\alpha,\beta)}(0)\right) \end{aligned}$$

$$= d_{(\alpha,\beta)}(0) + \alpha\left(2d_{(\alpha,\beta)}(x) - d_{(\alpha,\beta)}(0)\right)$$

$$= d_{(\alpha,\beta)}(0) + 2\alpha\left(d_{(\alpha,\beta)}(x)\right) - \alpha\left(d_{(\alpha,\beta)}(0)\right)$$

sehingga

$$2\alpha\left(d_{(\alpha,\beta)}(x)\right) = \alpha\left(d_{(\alpha,\beta)}(0)\right) - d_{(\alpha,\beta)}(0)$$

$$\alpha\left(d_{(\alpha,\beta)}(x)\right) = \frac{1}{2}\left(\alpha\left(d_{(\alpha,\beta)}(0)\right) - d_{(\alpha,\beta)}(0)\right)$$

$$\left(\alpha \circ d_{(\alpha,\beta)}\right)(x) = \frac{1}{2}\left(\left(\alpha \circ d_{(\alpha,\beta)}\right)(0) - d_{(\alpha,\beta)}(0)\right);\ \forall x \in L_P(X)$$

Proposisi 2.10 [11]

Misal $d_{(\alpha,\beta)}$ dan $d'_{(\alpha,\beta)}$ adalah dua(α,β)-Derivasi kiri pada *BCI*-Aljabar X. Jika $d_{(\alpha,\beta)} \circ d'_{(\alpha,\beta)} = 0$ pada $L_P(X)$, maka berlaku

$$\left(\alpha \circ d'_{(\alpha,\beta)}\right)(x) = \frac{1}{2}\left(\left(\alpha \circ d'_{(\alpha,\beta)}\right)(0) - d_{(\alpha,\beta)}(0)\right);\ \forall x \in L_P(X)$$

Bukti :

Misal $x \in L_P(X)$, dan $x + x \in L_P(X)$, sehingga $d'_{(\alpha,\beta)}(x + x) \in L_P(X)$. Dengan menggunakan proposisi 3.3.1 (1), proposisi 3.3.2 (1) dan (2) diperoleh

$$0 = \left(d_{(\alpha,\beta)} \circ d'_{(\alpha,\beta)}\right)(x + x)$$

$$= d_{(\alpha,\beta)}\left(d'_{(\alpha,\beta)}(x + x)\right)$$

$$= d_{(\alpha,\beta)}(0) + \alpha\left(d'_{(\alpha,\beta)}(x + x)\right)$$

$$= d_{(\alpha,\beta)}(0) + d'_{(\alpha,\beta)}(x + x)$$

$$= d_{(\alpha,\beta)}(0) + \left(d'_{(\alpha,\beta)}(x) + d'_{(\alpha,\beta)}(x) - d'_{(\alpha,\beta)}(0)\right)$$

$$= d_{(\alpha,\beta)}(0) + \alpha\left(2d'_{(\alpha,\beta)}(x) - d'_{(\alpha,\beta)}(0)\right)$$

$$= d_{(\alpha,\beta)}(0) + 2\alpha\left(d'_{(\alpha,\beta)}(x)\right) - \alpha\left(d'_{(\alpha,\beta)}(0)\right)$$

sehingga

$$2\alpha\left(d'_{(\alpha,\beta)}(x)\right) = \alpha\left(d'_{(\alpha,\beta)}(0)\right) - d_{(\alpha,\beta)}(0)$$

$$\alpha\left(d'_{(\alpha,\beta)}(x)\right) = \frac{1}{2}\left(\alpha\left(d'_{(\alpha,\beta)}(0)\right) - d_{(\alpha,\beta)}(0)\right)$$

$$\left(\alpha \circ d'_{(\alpha,\beta)}\right)(x) = \frac{1}{2}\left(\left(\alpha \circ d'_{(\alpha,\beta)}\right)(0) - d_{(\alpha,\beta)}(0)\right);\ \forall x \in L_P(X)$$

III. KESIMPULAN

Berdasarkan hasil pembahasan yang telah diuraikan, $d_{(\alpha,\beta)}$ adalah (α,β)-Derivasi kiri pada *BCI*-Aljabar X p-semi sederhana, maka $(\forall x, y \in X)\left(d_{(\alpha,\beta)}(x * y) = \alpha(x) * d_{(\alpha,\beta)}(y)\right)$. Jika ${d^2}_{(\alpha,\beta)} = 0$ pada $L_P(X)$, maka berlaku

$$\left(\alpha \circ d_{(\alpha,\beta)}\right)(x) = \frac{1}{2}\left(\left(\alpha \circ d_{(\alpha,\beta)}\right)(0) - d_{(\alpha,\beta)}(0)\right);\ \forall x \in L_P(X)$$

Misal $d_{(\alpha,\beta)}$ dan $d'_{(\alpha,\beta)}$ adalah dua(α,β)-Derivasi kiri pada *BCI*-Aljabar X. Jika $d_{(\alpha,\beta)} \circ d'_{(\alpha,\beta)} = 0$ pada $L_P(X)$, maka berlaku

$$\left(\alpha \circ d'_{(\alpha,\beta)}\right)(x) = \frac{1}{2}\left(\left(\alpha \circ d'_{(\alpha,\beta)}\right)(0) - d_{(\alpha,\beta)}(0)\right);\ \forall x \in L_P(X)$$

IV. DAFTAR PUSTAKA

[1] Abujabal, Hamza A.S. and Nora O. Al- Shehri. 2007. *On Left Derivation of BCI-Algebras*, Soochow Journal of Mathematics 33, 435-444.

[2] Al- Roqi, Abdullah M. 2014. *On Generalized (α,β)- Derivations in BCI- Algebras*. J. Appl. Math. & Informatics 32, 27-38.

[3] Gilbert, J. and Linda G. 1984. *Elements of Modern Algebra*. Prindle, Webel and Schmidt. Boston.

[4] Hidayanto, Erry. 2001. *StrukturAljabar*. UM Press: Malang.

[5] Howie, J. M. 1976. *An Introduction to Semigroup Theory*. Academic Press, London.

[6] Huang, Yisheng. 1999. *Characterizations of implicative BCI- Algebras*. Soochow Journal of Mathematics 25, 375-386.

[7] Javed, Malik Anjum and Muhammad Aslam. 2009. *A note on f- derivations of BCI-Algebras*. Commun. Korean Math 3, 321-331.

[8] Jun, Young Bae, Xiaolong Xin, and Eun Hwan Roh. 2004. *The role of atoms in BCI-Algebras*. Soochow Journal of Mathematics 30, 491-506.

[9] Lipschutz, Seymour. 1985. *Teori Himpunan*. Erlangga. Jakarta.

[10] Muhiuddin, G. and Abdullah M. Al- Roqi. 2012. *On (α,β) - Derivations in BCI-Algebras*. Hindawi Publishing Corporation Discrete Dynamics in Nature and Society 2012,pp 1-11.

[11] Muhiuddin, G. and Abdullah M. Al- Roqi. 2014. *On left (θ,ϕ) derivations in BCI-Algebras*. Journal of the Egyptian Mathematical Society 22, 157-161.

[12] Rum Wirastri,2009.*Derivasi BCI- Aljabar*.Tugas Akhir Jurusan Matematika FSM Universitas Diponegoro

[13] Tiande, L. and Xi Changchang. 1985. *p- Radical in BCI- Algebras*. Math. Japonica 30, 511-517.

[14] Zhan, Jianming and Yong Lin Liu. 2005. *On f- derivations of BCI- Algebras*. International Journal of Mathematics and Mathematical Sciences 2005, 1675-1684.