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OPTIMIZATION OF THE VARIATIONAL WEIGHTED FUZZY TIME SERIES MODEL USING PARTICLE SWARM OPTIMIZATION FOR TIME SERIES FORECASTING

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Abstract: Traditional Fuzzy Time Series and Weighted Fuzzy Time Series models fail to capture dynamics due to static interval partitioning. This study combines Variational Weighted Fuzzy Time Series (VWFTS) with the Particle Swarm Optimization (PSO) algorithm to automatically optimize intervals and improve accuracy against noise and uncertainty. Unlike conventional methods that use static weights, this approach introduces dynamic weight adjustments to address extreme data volatility. We applied the VWFTS-PSO model to three Indonesian datasets: the Jakarta Air Quality Index (AQI), monthly inflation, and gold prices. The results show that VWFTS-PSO consistently outperforms WFTS-PSO, as shown by lower error values across all evaluated datasets. For the Jakarta AQI data, VWFTS-PSO achieves a MAPE of 11.75% and an RMSE of 11.0475, outperforming WFTS-PSO, which records a MAPE of 13.83% and an RMSE of 13.1491. For the inflation data, VWFTS-PSO yields a MAPE of 7.68% and an RMSE of 0.5265, significantly lower than WFTS-PSO, with a MAPE of 10.47% and an RMSE of 0.7489. Furthermore, for the gold price dataset, the VWFTS-PSO model achieves a highly precise MAPE of 0.56%, outperforming the WFTS-PSO model (MAPE = 0.64%).

1. INTRODUCTION

Time series analysis, namely forecasting, is an important tool for decision-making based on available historical data. Various forecasting methods have been developed, including Fuzzy Time Series (FTS), introduced by Song and Chissom (1993). FTS forecasts linguistic data problems using fuzzy logic. FTS excels in handling uncertain and vague data. FTS also does not require normality or stationarity assumptions, making it flexible to use for various types of data (Muhammad et al., 2021).

However, FTS performance is sensitive to hyperparameters such as the number of partitions and the length of the partition intervals in the universe of discourse. Therefore, many modern researchers integrate FTS with optimization algorithms such as Particle Swarm Optimization (PSO). FTS has continued to develop, and in 2005, Yu introduced the concept of Weighted Fuzzy Time Series (WFTS). WFTS assigns different weights to each fuzzy relationship based on the data's chronological order, so more recent data has greater influence. However, this method still has limitations because it partitions the universe of discourse uniformly, which oversimplifies complex data distributions (Fan and Gou, 2023; Gupta & Kumar, 2023; Wang et al., 2024).

Conventional WFTS approaches also rely on static weighting mechanisms that fail to adequately capture dynamic fluctuations in state transition frequencies. To resolve these limitations, this study introduces a pioneering Variational Weighted Fuzzy Time Series with Particle Swarm Optimization (VWFTS-PSO) model. The Variational Weighted Fuzzy Time

Series (VWFTS) framework uses variational weighting, where the weight of each Fuzzy Logical Relationship (FLR) is determined adaptively based on its variation and temporal relevance. Thus, VWFTS effectively balances the frequency of occurrence of fuzzy relationships and their temporal proximity to the forecasting period. By integrating PSO into this framework, the proposed VWFTS-PSO approach adjusts weights based on variation in frequency and temporality while allowing partition interval lengths to vary non-uniformly within the universe of discourse. Ultimately, this joint optimization directly bridges existing methodological gaps and significantly enhances overall forecasting accuracy across complex real-world time series.

In this study, the combined VWFTS–PSO model is applied to evaluate its performance on three different types of time series data: the Jakarta Air Quality Index (AQI), Indonesia's monthly inflation data, and gold prices per gram in Indonesia. These three datasets were selected because they represent three important domains, namely:

a. Environment, through measurements of air quality in Jakarta, which tend to be volatile due to pollution and climate change;
b. Macroeconomics, through national inflation data that reflect the stability of prices for goods and services; and
c. Microfinance, through gold prices that reflect investment market behavior.

All three datasets exhibit complex, nonlinear, and dynamic characteristics, making them suitable for assessing the VWFTS–PSO model's ability to capture patterns and adaptively forecast future values. Accordingly, this study is expected to contribute to the development of fuzzy logic–based time series forecasting methods that are more accurate, adaptive, and efficient for various real-world applications.

2. LITERATURE REVIEW

According to Ujianto and Irawan (2015), forecasting is a process carried out when there is a time lag between actual data and the data to be predicted, to estimate uncertainty about events that may occur in the future. Forecasting is defined as the process of estimating future events based on past data. The success of a forecasting method highly depends on the accuracy of the data used. Box and Jenkins developed time series analysis and forecasting methods in 1970, particularly the ARIMA model. A time series is a sequence of data points ordered in time, typically observed at evenly spaced intervals (Prater et al., 2024). Time series data is a series of data measured or observed sequentially at regular or fixed time intervals. According to Montgomery et al. (2015), a time series is a set of observations generated sequentially over time.

A fuzzy set is an extension of a crisp set. Fuzzy sets allow one to work in uncertain and ambiguous situations and solve unexpected problems or problems with incomplete information (Davvaz et al., 2021).A fuzzy set S on the universe $X = \{x_1, x_2, x_3, \ldots, x_n\}$ is defined as a mathematical object of the form as given in Equation (1)

$$S = \{\langle x_i, \mu_S(x_i)\rangle; x_i \in X, i = 1,2,3,\ldots,n\} \quad (1)$$

Here, $\mu_S: X \to [0,1]$ denotes the membership function of the fuzzy set S, which assigns to each element X a degree of membership in (Fulara et al., 2026). A membership function is a curve that maps input data points to membership values (also called the degree of membership) between 0 and 1. Function-based approaches can be used to obtain membership values. There are several membership functions used in fuzzy set theory (Kusumadewi & Purnomo, 2010), namely:

i. Linear representation
ii. Triangular curve representation
iii. Trapezoidal representation

Prof. Lotfi Aliasker Zadeh first developed fuzzy logic in his 1965 paper on Fuzzy Set Theory. Fuzzy logic addresses uncertainty in the causes of a problem (Nasir & Suprianto, 2017). Fuzzy logic is widely used in part because it can tolerate imprecise or uncertain data. Fuzzification is a key process in fuzzy set theory that transforms crisp values into linguistic terms or fuzzy sets with membership degrees in the interval [0,1]. According to Kaulika et al. (2025), fuzzy set construction can use a triangular membership function, which better captures uncertainty in the data.

Song and Chissom proposed fuzzy time series (FTS) forecasting models that are particularly suitable for modeling imprecise, vague, or uncertain data, for which classical statistical techniques are inadequate (Fulara et al., 2026). According to Marzaen (2019), as cited by Muhammad (2021), fuzzy time series is a method that uses the basic concept of fuzzy sets as the foundation for calculations to determine forecast values for data to be predicted, in which the historical data are expressed in linguistic values.

A linguistic variable is a variable expressed using words that replace numerical values. Essentially, this forecasting model uses concepts from fuzzy set theory and fuzzy logic, which excel in modeling nonlinear and uncertain time series systems. According to Nikentari (2018), the fuzzy time series method transforms a set of real numbers into fuzzy sets by determining the bounds of the universe of discourse specified beforehand. Weighted Fuzzy Time Series (WFTS) is a development of the FTS method that addresses the previously ignored problem of repetition in fuzzy relations and assigns weights to each fuzzy relationship to reflect the varying importance of the sequence of fuzzy relationships (Yu, 2005). The algorithm used in WFTS differs from the traditional fuzzy time series method at the defuzzification stage. WFTS performs defuzzification by constructing weights for fuzzy sets, denoted as $A_{j1}, A_{j2}, \ldots, A_{jk}$ with corresponding weights $w_1, w_2, \ldots, w_k$. The weights are expressed in Equation (2).

$$w_{i^*} = \frac{w_i}{\sum_{h=1}^{k} w_h} \tag{2}$$

where w_i is the initial weight of the i-th fuzzy set, defined as $w_1 = 1$ and $w_i = c^{i-1}$ for $c \geq 1$ and $2 \leq i \leq k$, and h is the summation index. Then, we perform the vector transformation as expressed in Equation (3).

$$\boldsymbol{w}(t) = [w_1^*, w_2^*, \ldots, w_k^*] = \left[\frac{1}{\sum_{h=1}^{k} c^{h-1}}, \frac{c}{\sum_{h=1}^{k} c^{h-1}}, \ \ldots, \ \frac{c^{k-1}}{\sum_{h=1}^{k} c^{h-1}}\right] \tag{3}$$

where k denotes the number of fuzzy sets on the right-hand side of the relation (FLRG), or the next state, and I is the index of each fuzzy set ($i = 1,2,3, \ldots, k$).

According to Piotrowski et al. (2022), Particle Swarm Optimization (PSO) is a swarm-intelligence-based optimization method inspired by the behavior of bird flocks. Particle Swarm Optimization (PSO) is a global heuristic optimization technique introduced by Kennedy and Eberhart in 1995 (Achyani, 2018). The main idea of the PSO optimization algorithm is based on changes in velocity and acceleration for each particle, which moves toward the Pbest position and the Gbest position in the global version of the PSO algorithm. This random acceleration is modified independently in each direction (Slowik, 2020).

In each iteration of the PSO process, the fitness value of the current particle position is evaluated. The pbest and gbest values are then updated accordingly. At the end of each iteration, the velocity and position of each particle are updated according as expressed in Equation (4).

$$V_k^{t+1} = (w \times V_k^t) + (c_1 \times R_1 \times (P_{pbest_k} - X_k^t)) + (c_2 \times R_2 \times (P_{gbest_k} - X_k^t)) \quad (4)$$

$$X_k^{t+1} = (X_k^t + V_k^{t+1}) \quad (5)$$

with

V_k^{t+1} : velocity of particle (k) at iteration (t+1)
X_k^{t+1} : position of particle (k) at iteration (t+1)
P_{pbest_k} : pbest of particle (k) at iteration (t)
P_{gbest_k} : gbest of the swarm at iteration (t)
$c_1$dan c_2 : learning rates with values between 0 and 4
R_1 dan R_2 : uniformly distributed random numbers between 0 and 1
w : inertia weight

The accuracy of a forecasting model can be assessed by evaluating its performance. Typically, forecasting accuracy is assessed using error values. According to Chang et al. (2007), several measures can evaluate forecasting accuracy, such as MAPE and RMSE.

1) MAPE

MAPE is a measure of forecasting accuracy expressed as a percentage. MAPE is calculated by dividing the sum of absolute errors in each period by the actual observation value for that period. Then, it computes the average of these absolute percentage errors (Junianto, 2017). The MAPE formula is as expressed in Equation (6)

$$MAPE = \frac{100\%}{n} \sum_{i=1}^{n} \left| \frac{F_i - A_i}{F_i} \right| \quad (6)$$

n : number of data
F_i : actual value in the i-th period
A_i : forecast value in the i-th period

2) RMSE

Root Mean Square Error (RMSE) is the square root of the Mean Square Error (MSE). A forecasting model is better if its RMSE is smaller. The RMSE formula can be written mathematically as expressed in Equation (7).

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N} (F_i - A_i)^2}{n}} \quad (7)$$

n : number of data
F_i : actual value in the i-th period
A_i : forecast value in the i-th period

3. METHODOLOGY

This study uses three datasets: Jakarta AQI data, Indonesian inflation data, and gold prices per gram. Jakarta AQI data were obtained from the AQI: Real-time Air Quality Index website at https://www.aqi.in/. The site provides historical data on air pollution levels in various countries, including PM10 and PM2.5 levels at selected locations. Next, Indonesian inflation data were obtained from the Bank Indonesia (BI) website at https://www.bi.go.id/id/statistik/indikator/data-inflasi.aspx. This site provides periods and inflation rates for those periods. Gold price data per gram were obtained from the Pluang website at https://pluang.com/asset/gold, which provides historical gold price data for the last several years along with corresponding charts.

This study utilizes three datasets, including daily Air Quality Index data for Jakarta spanning three months from September 6, 2025 to November 19, 2025, monthly Indonesian inflation rate data over a 20-year period from January 2003 to August 2025, and daily gold

price per gram data covering a one-year period from October 18, 2024 to October 19, 2025. The Jakarta AQI dataset consists of 75 daily observations. The Indonesian inflation dataset consists of 272 observations with monthly frequency. The gold price per gram dataset consists of 362 observations with daily frequency.

Variational Weighted Fuzzy Time Series (VWFTS) extends the Weighted Fuzzy Time Series (WFTS) method by adding a variational weighting mechanism to form fuzzy logical relationships. The main difference lies in how it assigns weights. While WFTS considers only the recency of past data, VWFTS accounts for both frequency and recency.

According to Rozy et al. (2021), integrating variational weights into FTS has been shown to improve forecasting performance, especially when combined with optimization algorithms such as Differential Evolution (DE) or Particle Swarm Optimization (PSO). In Variational Weighted Fuzzy Time Series (VWFTS), fuzzy relationship weights are determined linearly based on the chronological order of pattern occurrences. More recent patterns are given higher weights than older patterns, so that the model still considers the entire data history (historical frequency) but emphasizes the relevance of the most recent data (recency).

Fuzzy Logical Relationships (FLRs) are constructed from data that have already been fuzzified and then grouped. Unlike traditional methods that group the current state and collect a set of next states for each current state, this approach uses a linear weighting scheme in which the next states are arranged chronologically in a sequence.

In this sequence, repetitions may occur, and this is allowed. Let C be the current state, and A_i be a linguistic variable (category representing a group) that represents the next state; if i < j, then A_i is considered to occur earlier chronologically than A_j.

$$C \rightarrow A_1, A_2, \ldots, \text{with } A_i \in U \tag{8}$$

Weights are then assigned to each Fuzzy Logical Relationship Group (FLRG) linearly, starting at 1 and incrementing by 1 until all next states are counted. These weights are then normalized by dividing each weight by the total sum of all weights. The linear weighting formula is given as expressed in Equation (9).

$$w_i = \frac{2i}{m(m+1)} \tag{9}$$

w_i : the weight value assigned to the i-th next state.
i : the position sequence/index of the next state (where $i = 1,2, \ldots, m$)
m : the total number of next states contained within a Fuzzy Logical Relationship Group (FLRG).

With FLRGs constructed and weights determined, forecasting is performed by examining the fuzzy position of the current data. Referring to the FLRG, the forecast result is obtained by computing a variational weighted average of the midpoints of the linguistic variable partitions contained in the FLRG. Mathematically, the forecasting calculation using the VWFTS method is as follows Equation (10).

$$y(t+1) = \sum(w_i \times M(A_i)) \tag{10}$$

where $y(t+1)$ is the forecasted value for the next time period, t is the index of the current time period, w_i is the normalized weight for the i-th fuzzy set or relationship, and $M(A_i)$ is the midpoint of the partition whose fuzzy variable u_j (a value in the UoD interval)

in A_i has the maximum membership value of 1. The value $y(t+1)$ is obtained by summing the products of w_i and $M(A_i)$ as a form of defuzzification to obtain a crisp predicted value.

The performance of Fuzzy Time Series (FTS) methods, including VWFTS, is highly sensitive to the partitioning of the universe of discourse (UoD). Manually determining partitions is often inefficient and unlikely to reach an optimal configuration, which can reduce forecasting accuracy. To address this challenge, Particle Swarm Optimization (PSO) is integrated as a solution-search mechanism. In this hybrid architecture, PSO, a metaheuristic optimization algorithm, explores the solution space to find the most effective UoD partition combinations. Each particle in the PSO algorithm represents one candidate solution, specifically a complete set of partition intervals.

The proposed VWFTS-PSO methodology aims to improve VWFTS forecasting accuracy by optimizing its fundamental problem: how to partition the universe of discourse (UoD). PSO is used to search for the best partition set. Mathematically, to construct n_p partitions, the algorithm must find $n_p - 1$ cutting points (positions) between the minimum (D_{min}) and maximum (D_{max}) values of the data. Each PSO particle is a complete candidate solution, where its position in the search space ($n_p - 1$ dimensions) represents a complete proposed set of partitions.

The partitioning stage of the universe of discourse (U) is a fundamental step in the Variational Weighted Fuzzy Time Series (VWFTS) method. In this study, partitioning is performed dynamically using Particle Swarm Optimization (PSO) through two main mechanisms: determining the number of intervals (n_p) and constructing variable-length interval boundaries through a decoding process. The VWFTS-PSO procedure consists of nine steps: Step 1, determining the universe of discourse; Step 2, determining the number and lengths of intervals through particle initialization, sorting, and interval length determination; Step 3, defining fuzzy sets; Step 4, performing fuzzification; Step 5, constructing fuzzy logic relations (FLRs); Step 6, constructing variational weighted fuzzy logic relationship groups (FLRGs); Step 7, assigning weights; Step 8, performing defuzzification; and Step 9, generating forecasts.

The optimization process runs in cycles (iterations) in step 2. At each iteration, every PSO particle calls VWFTS as a subroutine to evaluate the partition set it proposes. VWFTS builds the model and returns the forecast error, which PSO uses as the fitness value. Each particle then updates its own best solution record ($P^{p}_{\beta est}$), and the entire swarm updates the global best solution record ($P^{g}_{\beta est}$) found so far. Based on this information (individual and swarm experience), each particle adjusts its velocity and position (according to Equations 4 and 5), gradually moving toward the solution that yields the smallest error.

The success of this algorithm depends on the input-parameter configuration. Historical data (X and y) and its length (n) form the foundation for forecasting. The parameter m (number of fuzzy sets) determines partition granularity, while p (number of particles) controls how broadly the search space is explored. The bounds D_{min} and D_{max} ensure the search remains within a valid range, and max iter (maximum iterations) determines when the optimization stops. Together, these parameters guide PSO to find the most effective UoD partitioning scheme, thereby maximizing VWFTS accuracy.

To provide an overview of the technical steps carried out in the PSO optimization of the VWFTS model in this study, Figure 1 presents a flowchart of the overall VWFTS-PSO workflow.

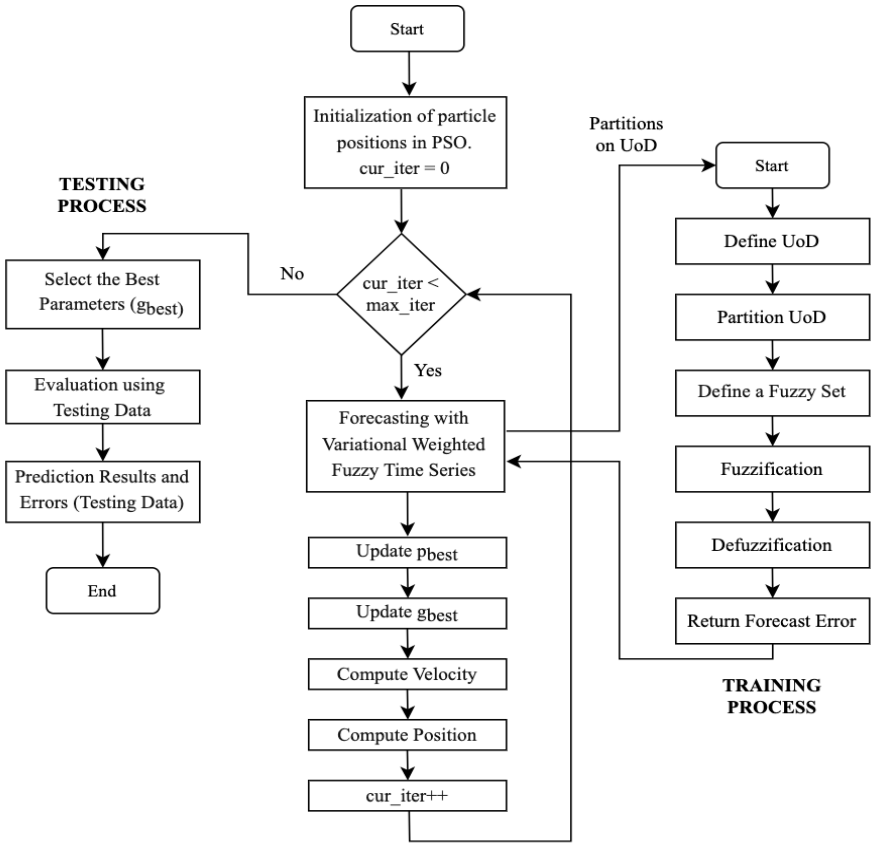


Figure 1. flowchart

4. RESULT

In this study, we constructed a training dataset to train the model and used a test dataset to evaluate the model built from the training data. The implementation of the Variational Weighted Fuzzy Time Series with Particle Swarm Optimization (VWFTS-PSO), which is an extension of Yu's WFTS, was modified by allowing the partition interval lengths to vary (non-uniform). In addition, to form the weights used in the defuzzification process, a linear weighting scheme was applied that accounts for both the frequency and recency of the data. The historical data for Jakarta AQI, Indonesia's inflation, and gold prices were partitioned into a 70% training set and a 30% testing set to determine the optimal model. In this study, the parameter settings for the VWFTS-PSO model are configured as c_1 = 1.496, c_2 = 1.496, w_{max} = 0.9, and w_{min}= 0.4.

Table 1. Comparison of the Optimal Number of Partitions and Length in the VWFTS-PSO and WFTS with Training and Testing Dataset

Dataset	Optimal Number of Partitions	
	VWFTS-PSO	WFTS-PSO
Jakarta AQI	14	20
Inflation	149	46
Gold Price	159	133

The forecasting accuracy measures reported in Table 2 and Table 3 are evaluated based on the testing dataset (out-of-sample) to assess the true predictive performance of the proposed model.

Table 2. Comparison of the RMSE and MAPE in the VWFTS-PSO and WFTS (Training Dataset)

Dataset	VWFTS-PSO		WFTS-PSO	
	RMSE	MAPE	RMSE	MAPE
Jakarta AQI	10.2262	9.84%	9.455	9.02%
Inflation	0.5584	6.31%	0.8254	8.36%
Gold Price	11321.7238	0.45%	13168.4081	0.47%

Based on Table 2, the VWFTS-PSO model demonstrates exceptional forecasting performance, outperforming WFTS-PSO across most training datasets. Specifically, VWFTS-PSO achieves significantly lower error rates for the Inflation dataset (RMSE of 0.5584 and MAPE of 6.31% versus 0.8254 and 8.36%) and the Gold Price dataset (RMSE of 11321.7238 and MAPE of 0.45% versus 13168.4081 and 0.47%), highlighting its superior absolute precision and relative accuracy in complex predictive scenarios.

Table 3. Comparison of the RMSE and MAPE in the VWFTS-PSO and WFTS (Testing Dataset)

Dataset	VWFTS-PSO		WFTS-PSO	
	RMSE	MAPE	RMSE	MAPE
Jakarta AQI	17.0144	17.27%	17.7739	19.61%
Inflation	0.4084	24.84%	0.4416	25.62%
Gold Price	303460.8311	4.85%	283035.8678	4.56%

Based on Table 3, the proposed VWFTS-PSO model demonstrates superior overall forecasting performance on the testing dataset, outperforming WFTS-PSO across most evaluated metrics. Specifically, VWFTS-PSO performs significantly better on the Jakarta AQI dataset, with lower error rates (RMSE of 17.0144 and MAPE of 17.27% compared to 17.7739 and 19.61%), and on the Inflation dataset (RMSE of 0.4084 and MAPE of 24.84% compared to 0.4416 and 25.62%). Although WFTS-PSO yields slightly lower errors for the Gold Price dataset, VWFTS-PSO remains highly competitive with a low MAPE of 4.85%, underscoring its robust generalization and reliable forecasting ability on unseen test data.

After determining the optimal number of partitions (n) based on the testing dataset evaluation (as shown in Table 1), the best model was applied to the entire dataset to visualize the overall forecasting performance and to forecast future periods. To illustrate the implementation process, the step-by-step forecasting process using the VWFTS-PSO model for the Jakarta AQI dataset is outlined below.

Step 1. Determining the Universe of Discourse

The universe of discourse U is determined using the formula $U = [D_{min} - D_1;\ D_{max} - D_2]$, yielding the universe of discourse that covers the historical Jakarta AQI data as follows.

$$U = [25.3\ ;\ 141.5]$$

Step 2. Determining the Number and Length of Intervals

Based on Particle Swarm Optimization (PSO), the optimal number of partitions for the Jakarta AQI dataset is 14, with interval boundaries ranging from 25.30 to 141.70. For example, the first two intervals are $\mu_1 = [25.30,35.56]$ and $\mu_2 = [35.56,42.96]$, with corresponding midpoints of 30.43 and 39.26, respectively. The midpoint of the final interval is 131.60. These midpoints are subsequently used in the fuzzification and forecasting processes. The optimal number of partitions was determined using Particle Swarm Optimization (PSO), which evaluates candidate partitioning schemes based on a forecasting error criterion defined as the fitness function. The partitioning scheme with the lowest fitness value was selected as the optimal solution. Based on this criterion, PSO identified 14 partitions for the Jakarta AQI dataset, with interval boundaries ranging from 25.30 to 141.70. The midpoint of each interval was then calculated for the subsequent fuzzification and forecasting processes.

Step 3. Defining the Fuzzy Sets

The PSO optimization results yield 14 fuzzy sets, as follows.

$$A_1 = \frac{1}{u_1} + \frac{0.5}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \cdots + \frac{0}{u_{10}} + \frac{0}{u_{11}} + \frac{0}{u_{12}} + \frac{0}{u_{13}} + \frac{0}{u_{14}}$$

$$A_2 = \frac{0.5}{u_1} + \frac{1}{u_2} + \frac{0.5}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \cdots + \frac{0}{u_{10}} + \frac{0}{u_{11}} + \frac{0}{u_{12}} + \frac{0}{u_{13}} + \frac{0}{u_{14}}$$

$$\cdots$$

$$A_1 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \cdots + \frac{0}{u_{10}} + \frac{0}{u_{11}} + \frac{0}{u_{12}} + \frac{0.5}{u_{13}} + \frac{1}{u_{14}}$$

Step 4. Fuzzification

If the time series data falls within the interval u_n, the data will be fuzzified as A_i. The fuzzification results of the Jakarta AQI data are presented Table 4.

Table 4. Fuzzification

Date	AQI Data	Fuzzification
September 6, 2025	42	A_2
September 7, 2025	44	A_3
…	…	…
November 19, 2025	91	A_8

Step 5. Constructing Fuzzy Logic Relations (FLRs)

Based on the previous fuzzification results, determine the FLRs by connecting consecutive linguistic values. The resulting FLRs are presented Table 5.

Table 5. FLR

Date	AQI Data	Current State	Next State	FLR
September 6, 2025	42	A_2	A_3	$A_2 \rightarrow A_3$
September 7, 2025	44	A_3	A_5	$A_3 \rightarrow A_5$
…	…	…	…	…
November 19, 2025	91	A_8	*	*

Step 6. Constructing Variational Weighted Fuzzy Logic Relationship Groups

All Fuzzy Logic Relationships (FLRs) are grouped to form state transition groups. The resulting FLRG for the Jakarta AQI dataset is as presented Table 6.

Table 6. FLRG

CS	Next State	FLRG
A_1	A_3	$A_1 \rightarrow A_3$
A_2	A_3, A_2, A_1, A_3	$A_2 \rightarrow A_3, A_2, A_1, A_3$
…	…	…
A_{14}	$A_{14}, A_{14}, A_{12}, A_{11}, A_{12}, A_{13}$	$A_{14} \rightarrow A_{14}, A_{14}, A_{12}, A_{11}, A_{12}, A_{13}$

Step 7. Assigning Weights

Weights are assigned to each next state in the Fuzzy Logical Relationship Group (FLRG), where (m) represents the number of next-state occurrences. For example, the FLRG $A_2 \rightarrow A_3, A_2, A_1, A_3$, with (m=4), produces normalized weights of 0.1, 0.2, 0.3, and 0.4, respectively. These weights represent the relative contributions of the next states in the forecasting process.

Step 8. Defuzzification and Forecasting Results

Forecasting is performed using the constructed FLRGs and their assigned weights based on the current fuzzy state. For the Jakarta AQI dataset, the model produces forecasts of 40.68 and 79.26 for September 7 and November 19, 2025, respectively, compared with actual

values of 44 and 91. The same procedure is applied to the Indonesian Inflation and Gold Price datasets. The forecasting results for all three datasets are illustrated in the following figures.

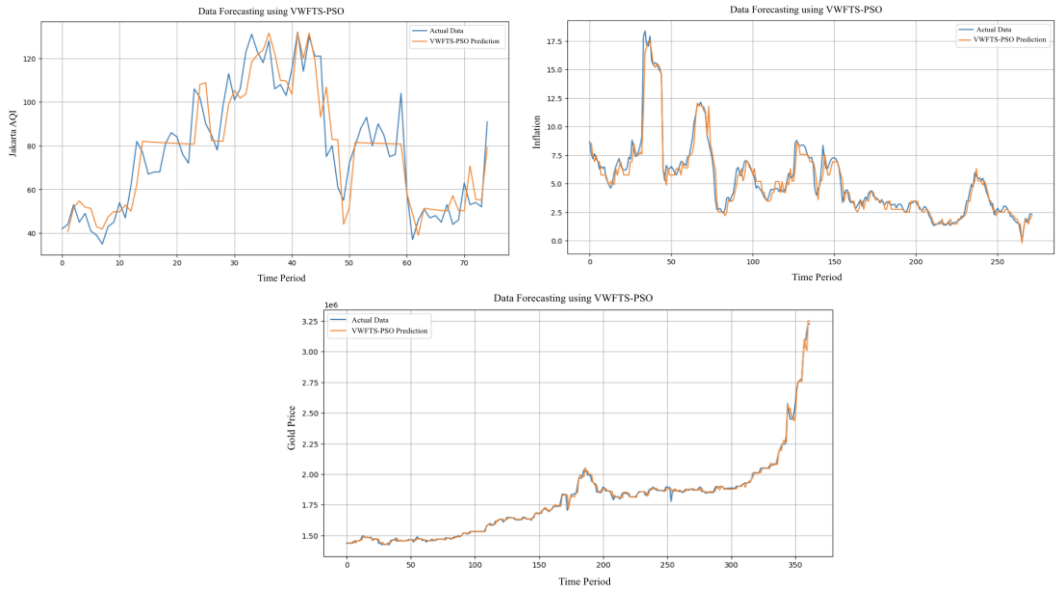


Figure 2. Forecasting using VWFTS-PSO: (a) Jakarta AQI; (b) Inflation; (c) Gold Price

Next, forecasting is also carried out using the WFTS-PSO method, and the resulting forecasting plot using WFTS-PSO is obtained for comparison with the VWFTS-PSO method, as follows.

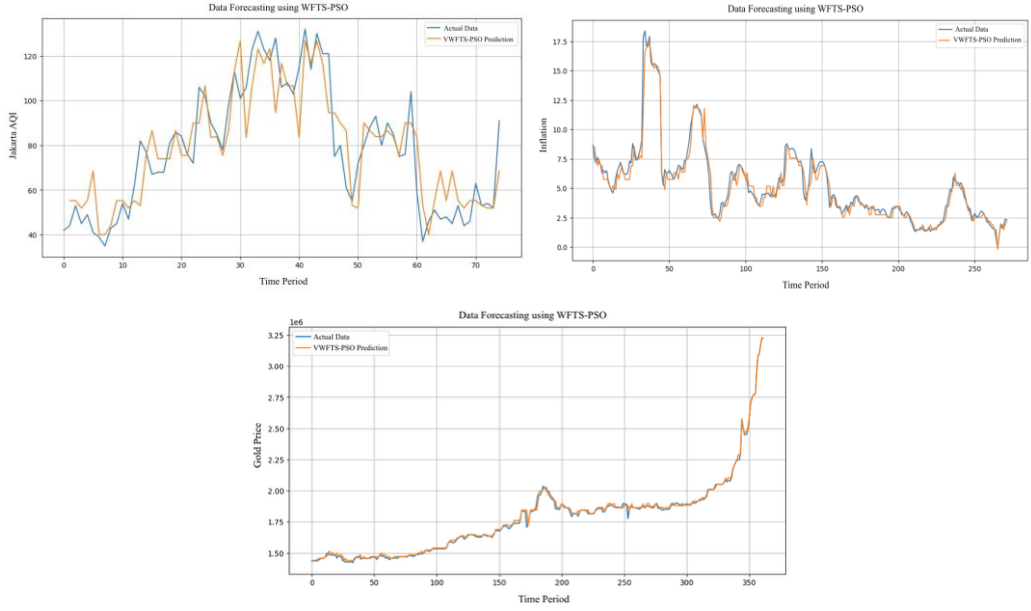


Figure 3. Forecasting using WFTS-PSO: (a) Jakarta AQI; (b) Inflation; (c) Gold Price

Based on the forecasting results, the VWFTS-PSO method generally demonstrates better forecasting performance than WFTS-PSO across the three datasets. For Jakarta AQI, VWFTS-PSO achieves an RMSE of 11.0475 and a MAPE of 11.75%, compared with 13.1491 and 13.83% for WFTS-PSO. Similarly, for inflation, VWFTS-PSO reduces RMSE from 0.7489 to 0.5265 and MAPE from 10.47% to 7.68%. For gold prices, VWFTS-PSO achieves a lower MAPE (0.56% versus 0.64%), although its RMSE is higher (20,081.2408

versus 17,093.5497). Overall, these findings indicate that PSO-optimized variational weights improve percentage-based forecasting accuracy across all three datasets, although improvements in RMSE are not consistent.

5. CONCLUSIONS

Based on the analysis and discussion presented in the previous chapter regarding the application of the Variational Weighted Fuzzy Time Series model based on Particle Swarm Optimization (VWFTS-PSO), several conclusions can be drawn as follows.

a. The VWFTS-PSO model consistently demonstrates superior forecasting performance compared to the conventional WFTS-PSO model across the two evaluated datasets for MAPE: Jakarta AQI and Indonesian Inflation.
b. For the Jakarta AQI data, the VWFTS-PSO model achieves better accuracy with a MAPE of 11.75% and an RMSE of 11.0475, outperforming the WFTS-PSO model, which has a MAPE of 13.83% and an RMSE of 13.1491.
c. A significant improvement is also observed in the Indonesian Inflation data, where the VWFTS-PSO model produces a MAPE of 7.68% and an RMSE of 0.5265. These values are much lower than those of the WFTS-PSO method, which yields a MAPE of 10.47% and an RMSE of 0.7489.
d. For the Gold Price data, the VWFTS-PSO model is able to achieve a highly precise level of accuracy with the lowest MAPE of 0.56%, demonstrating that this model is highly effective for predicting data with large value scales but dynamic daily fluctuations.

Overall, the combination of variational weighting and PSO optimization has been shown to reduce prediction errors and provide more representative forecasting results than the baseline model.

REFERENCES

Achyani, Y.E. 2018. Penerapan Metode Particle Swarm Optimization Pada Optimasi Prediksi Pemasaran Langsung. *Jurnal Informatika*, Volume 5(1), pp. 1–11.

Chang, P.-C., Liu, C.-H., Yeh, C.-H. dan Chen, S.-H. 2006. The Development of a Weighted Evolving Fuzzy Neural Network. *Lecture Notes in Artificial Intelligence*, Volume 4114, pp. 1228–1237.

Davvaz, B., Mukhlash, I. dan Soleha, S. 2021. Himpunan Fuzzy dan Rough Sets. *Limits: Journal of Mathematics and Its Applications*, [Online] Volume 18(1), pp. 79. https://doi.org/10.12962/limits.v18i1.7705.

Fan, Z. dan Gou, J. 2023. Predicting body fat using a novel fuzzy-weighted approach optimized by the whale optimization algorithm. *Expert Systems with Applications*, [Online] Volume 217. https://doi.org/10.1016/j.eswa.2023.119558.

Fulara, S.P., Tak, N. dan Kumar, S. 2026. Time series forecasting method based on complete ensemble empirical mode decomposition with adaptive noise and hesitant fuzzy cognitive map. *Applied Soft Computing*, [Online] Volume 198. https://doi.org/10.1016/j.asoc.2026.115282.

Gupta, K.K. dan Kumar, S. 2023. K-Means Clustering Based High Order Weighted Probabilistic Fuzzy Time Series Forecasting Method. *Cybernetics and Systems*, [Online] Volume 54(2), pp. 197–219. https://doi.org/10.1080/01969722.2022.2058691.

Ihsanuddin, A. D., Ispriyanti, D., and Tarno, T., 2023. PENERAPAN METODE FUZZY TIME SERIES MENGGUNAKAN PARTICLE SWARM OPTIMIZATION ALGORITHM UNTUK PERAMALAN INDEKS SAHAM LQ45. *Jurnal Gaussian*, [Online] Volume 12(1), pp. 10-19. https://doi.org/10.14710/j.gauss.12.1.10-19.

Junianto, M.B.S. 2017. FUZZY INFERENCE SYSTEM MAMDANI DAN THE MEAN ABSOLUTE PERCENTAGE ERROR (MAPE) UNTUK PREDIKSI PERMINTAAN DOMPET PULSA PADA XL AXIATA DEPOK. *Jurnal Universitas Pamulang*, Volume 2(2), pp. 97–102.

Kaulika, F.N., Yulianti, K., Puspita, E. dan Marwati, R. 2025. PERAMALAN KUALITAS UDARA KOTA BANDUNG DENGAN METODE FUZZY MAMDANI DAN ALGORITMA GENETIKA-FUZZY TIME SERIES. *Epsilon: Jurnal Matematika Murni dan Terapan*, [Online] Volume 19(1), pp. 41-53. https://ppjp.ulm.ac.id/journals/index.php/epsilon/article/view/1489.

Muhammad, M., Wahyuningsih, S. dan Siringoringo, M. 2021. Peramalan Nilai Tukar Petani Subsektor Peternakan Menggunakan Fuzzy Time Series Lee. *Jambura Journal of Mathematics*, [Online] Volume 3(1), pp. 1–15. https://doi.org/10.34312/jjom.v3i1.5940.

Montgomery, D.C., Jennings, C.L. dan Kulahci, M. 2015. *Introduction to Time Series Analysis and Forecasting*. Edisi ke-2. Hoboken, NJ: John Wiley & Sons.

Nasir, J. dan Suprianto, J. 2017. ANALISIS FUZZY LOGIC MENENTUKAN PEMILIHAN MOTOR HONDA DENGAN METODE MAMDANI. *Jurnal Edik Informatika*, [Online] Volume 3(2), pp. 177–186. https://doi.org/10.22202/jei.2017.v3i2.1962.

Nikentari, N., Bettiza, M. dan Pratiwi, H.S. 2018. Prediksi Kecepatan Angin Menggunakan Adaptive Neuro Fuzzy (ANFIS) dan Radial Basis Function Neural Network (RBFNN). *JEPIN (Jurnal Edukasi dan Penelitian Informatika)*, Volume 4(1), pp. 70–75.

Piotrowski, A.P., Napiorkowski, J.J. dan Piotrowska, A.E. 2020. Population size in Particle Swarm Optimization. *Swarm and Evolutionary Computation*, [Online] Volume 58. https://doi.org/10.1016/j.swevo.2020.100718.

Prater, R., Hanne, T. dan Dornberger, R. 2024. Generalized Performance of LSTM in Time-Series Forecasting. *Applied Artificial Intelligence*, [Online] Volume 38(1). https://doi.org/10.1080/08839514.2024.2377510.

Pribadi, D., Saputra, R.A., Hudin, J.M. dan Gunawan. 2020. *Sistem Pendukung Keputusan*. Yogyakarta: Graha Ilmu.

Rozy, A.F., Solimun, S. dan Wardhani, N.W.S. 2023. Peningkatan Akurasi Metode Weighted Fuzzy Time Series Forecasting Menggunakan Algoritma Evolusi Differensial dan Fuzzy C-Means. *Jurnal Teknologi Informasi dan Ilmu Komputer*, [Online] Volume 10(5), pp. 1047–1054. https://doi.org/10.25126/jtiik.2023107505.

Słowik, A. 2020. *Swarm Intelligence Algorithms: A Tutorial*. Boca Raton, FL: CRC Press.

Ujianto, Y. dan Irawan, M.I. 2015. Perbandingan Performansi Metode Peramalan Fuzzy Time Series yang Dimodifikasi dan Jaringan Syaraf Tiruan Backpropagation (Studi Kasus: Penutupan Harga IHSG). *Jurnal Sains dan Senin ITS,* Volume 4.

Wang, B., Liu, X., Chi, M. dan Li, Y. 2024. Bayesian network based probabilistic weighted high-order fuzzy time series forecasting. *Expert Systems with Applications*, [Online] Volume 237. https://doi.org/10.1016/j.eswa.2023.121430.

Yu, H.K. 2005. Weighted fuzzy time series models for TAIEX forecasting. *Physica A: Statistical Mechanics and its Applications*, [Online] Volume 349(3–4), pp. 609–624. Available at: https://doi.org/10.1016/j.physa.2004.11.006.

Zoghi, M., Ehsani, A.H., Sadat, M., Amiri, M.J. dan Karimi, S. 2017. Optimization solar site selection by fuzzy logic model and weighted linear combination method in arid and semi-arid region: A case study Isfahan-IRAN. *Renewable and Sustainable Energy Reviews*, [Online] Volume 68, pp. 986–995. https://doi.org/10.1016/j.rser.2015.07.014.